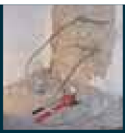
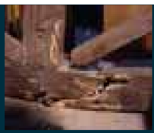
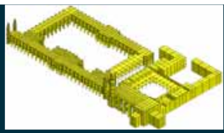
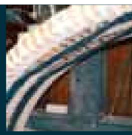
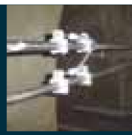
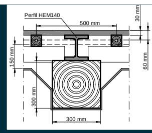
     

STRENGTH OF MATERIALS - I

CE-205

Dr. Abdul Qadir Bhatti

E-mail : draqbhatti@gmail.com, bhatti-nit@nust.edu.pk

National Institute of Civil Engineering (NICE)

NUST School of Civil & Environmental Engineering

Islamabad, Pakistan

School of Civil and Environmental Engineering (SCEE), National University of Sciences & Technology, NUST, Islamabad, Pakistan

Shear Stress

- Shear stresses are produced by equal and opposite parallel forces not in line.
- The forces tend to make one part of the material slide over the other part.
- Shear stress is tangential to the area over which it acts.

Problem 117

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1-11b if P = 400 kN. The shearing strength of the bolt is 300 MPa.

Solution 117

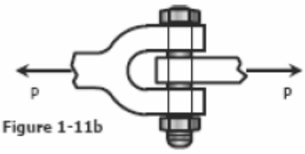


Figure 1-11b

The bolt is subject to double shear.

$V = \tau A$

$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

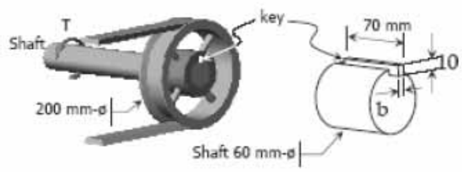
$d = 29.13\text{ mm}$

Problem 118

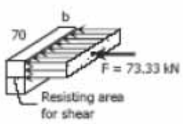
A 200-mm-diameter pulley is prevented from rotating relative to 60-mm-diameter shaft by a 70-mm-long key, as shown in Fig. P-118. If a torque T = 2.2 kN·m is applied to the shaft, determine the width b if the

allowable shearing stress in the key is 60 MPa.

Figure P-118



Solution 118



FBD of Pin

$T = 0.03F$

$2.2 = 0.03F$

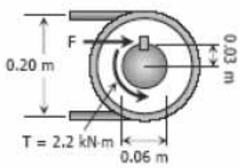
$F = 73.33\text{ kN}$

$V = \tau A$

Where: $V = F = 73.33\text{ kN}$
$A = 70b$; $\tau = 60\text{ MPa}$

$73.33(1000) = 60(70b)$

$b = 17.46\text{ mm}$



Problem 119

Compute the shearing stress in the pin at B for the member supported as shown in Fig. P-119. The pin diameter is 20 mm.

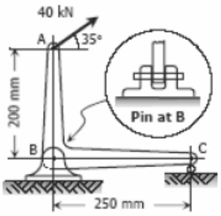
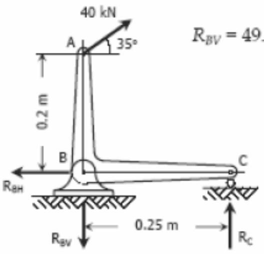


Figure P-119

Solution 119



Free Body Diagram

From the FBD:

$\Sigma M_C = 0$

$0.25 R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$

$= 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

$= 59.076$ kN → shear force of pin at B

$V_B = \tau_B A$ → double shear

$59.076\,(1000) = \tau_B\,\{2[\frac{1}{4}\pi(20^2)]\}$

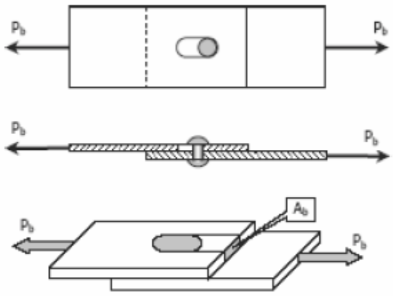
$\tau_B = 94.02$ MPa

HW#2:Try On Your Own

- Problem 120, 121, 122, 123
- Tex Book : Strength of Materials By Andrew Pytel and Singer

Bearing Stress

Bearing stress is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.



$$\sigma_b = \frac{P_b}{A_b}$$

SOLVED PROBLEMS IN BEARING STRESS

Problem 125

In Fig. 1-12, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.

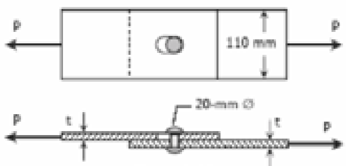


Figure 1-12

Solution 125

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ mm

(b) Largest average tensile stress in the plate:

$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ MPa

Problem 128

Shape of beam = W18 × 86
Shape of girder = W24 × 117
Shape of angles = 4 × 3-½ × 3/8
Diameter of rivets = 7/8 inch
Allowable shear stress = 15 ksi
Allowable bearing stress = 32 ksi

Required: Allowable load on the connection

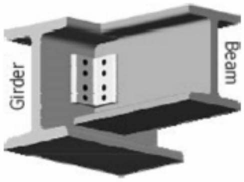


Figure 1-13

Solution 128
Relevant data from the table (Appendix B of textbook): Properties of Wide-Flange Sections (W shapes): U.S. Customary Units

Designation	Web thickness
W18 × 86	0.480 in
W24 × 117	0.550 in

Shearing strength of rivets:
There are 8 single-shear rivets in the girder and 4 double-shear (equivalent to 8 single-shear) in the beam, thus, the shear strength of rivets in girder and beam are equal.

$$V = \tau A = 15[\,\tfrac{1}{4}\pi(\tfrac{7}{8})^2(8)\,]$$
$$V = 72.16\,\text{kips}$$

Bearing strength on the girder:
The thickness of girder W24 × 117 is 0.550 inch while that of the angle clip $L4 \times 3\frac{1}{2} \times \frac{3}{8}$ is $\frac{3}{8}$ or 0.375 inch, thus, the critical in bearing is the clip.

$$P = \sigma_b A_b = 32[\,\tfrac{7}{8}(0.375)(8)\,]$$
$$P = 84\,\text{kips}$$

Bearing strength on the beam:
The thickness of beam W18 × 86 is 0.480 inch while that of the clip angle is 2 × 0.375 = 0.75 inch (clip angles are on both sides of the beam), thus, the critical in bearing is the beam.

$$P = \sigma_b A_b = 32[\,\tfrac{7}{8}(0.480)(4)\,]$$
$$P = 53.76\,\text{kips}$$

The allowable load on the connection is $P = 53.76\,\text{kips}$ → ***answer***

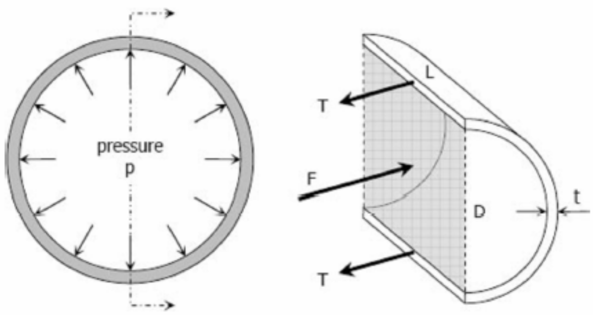
HW#4:Try On Your Own

- Problem 127, 128, 129
- Tex Book : Strength of Materials By Andrew Pytel and Singer

Thin-walled Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

TANGENTIAL STRESS (Circumferential Stress)
Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



The forces acting are the total pressures caused by the internal pressure p and the total tension in the walls T.

$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$

$$\Sigma F_H = 0$$
$$F = 2T$$

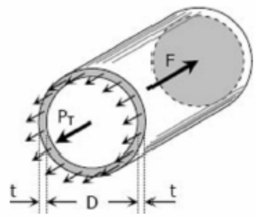
$pDL = 2(\sigma_t tL)$

$$\sigma_t = \frac{pD}{2t}$$

If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

LONGITUDINAL STRESS, σ_L
Consider the free body diagram in the transverse section of the tank:



The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to πDt

$F = pA = p\,\frac{\pi}{4}D^2$
$P_T = \sigma_L \pi Dt$
$\Sigma F_H = 0$
$P_T = F$
$\sigma_L \pi Dt = p\,\frac{\pi}{4}D^2$

$$\sigma_t = \frac{pD}{4t}$$

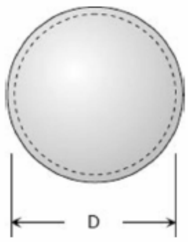
If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\sigma_L$$

SPHERICAL SHELL



If a spherical tank of diameter D and thickness t contains gas under a pressure of p, the stress at the wall can be expressed as:

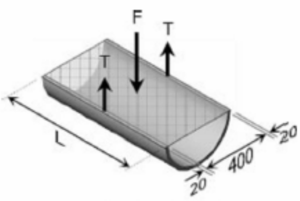
$$\sigma_t = \frac{(p_i - p_o)D}{4t}$$

Given:
Diameter of cylindrical pressure vessel = 400 mm
Wall thickness = 20 mm
Internal pressure = 4.5 MN/m^2
Allowable stress = 120 MN/m^2

Required:
Longitudinal stress
Tangential stress
Maximum amount of internal pressure that can be applied
Expected fracture if failure occurs

Solution 133

Part (a)
Tangential stress (longitudinal section):



Longitudinal Section

$$F = 2T$$

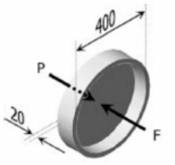
$$pDL = 2(\sigma_t t L)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

Problem 133

$\sigma_t = 45\text{ MPa}$ → *answer*

Longitudinal Stress (transverse section):



Transverse Section

$F = P$

$\frac{1}{4}\pi D^2 p = \sigma_l(\pi D t)$

$\sigma_l = \dfrac{pD}{4t} = \dfrac{4.5(400)}{4(20)}$

$\sigma_l = 22.5\text{ MPa}$ → *answer*

Part (b)

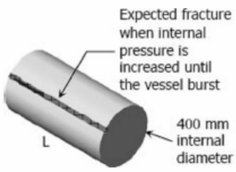
From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

$\sigma_t = \dfrac{pD}{2t}$

$120 = \dfrac{p(400)}{2(20)}$

$p = 12\text{ MPa}$ → *answer*

The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.

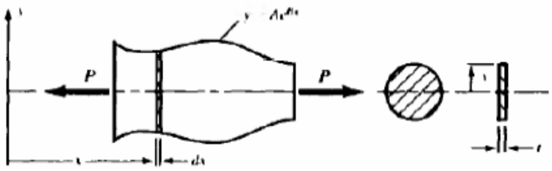


HW#4:Try On Your Own

- Problem 135, 137
- Tex Book : Strength of Materials By Andrew Pytel and Singer

LABORATORY

An elastic bar of variable cross section is loaded by axial tension or compression at its ends as shown in Fig. 1-31. The variation of cross-sectional dimension may be known either analytically or numerically along the dimension in the axial direction. Write a FORTRAN program for change of length of the bar for the cases of (*a*) a bar of solid circular cross section and (*b*) a flat slab of constant thickness *t* as shown in Figs. 1-31(*b*) and 1-31(*c*), respectively. The contour of the bar is described by the equation $y = Ae^{Bx}$, where x is the axial coordinate.



(*a*) (*b*) (*c*)

1-31(*a*). The cross-sectional area of each such subsegment is taken to be constant and we then apply the relation

$$\Delta = \frac{PL}{AE}$$

to this segment, where the length of the segment is *dx* and *A* is the cross-sectional area of the segment. Clearly *A* may be found if the equation $y = y(x)$ for the cross section is known, or, alternatively, measurements may be made at a number of stations along the length of the bar and the area found numerically at each such station.

This approach is represented by the following FORTRAN program which is self-prompting. Tensile loadings are regarded as positive and compressives as negative.

Note that in the equation describing the shape of the bar, $y = Ae^{Bx}$, *e* represents the base of natural logs, and *A* and *B* are parameters of the contour. Note in particular that this *A* is *not* cross-sectional area.


```
************************************************************************
                     PROGRAM SLBTEN2(INPUT,OUTPUT)
************************************************************************
*
*          AUTHOR: KATHLEEN DERWIN
*          DATE  : FEBRUARY 5, 1989
*
*    BRIEF DESCRIPTION:
*       THIS PROGRAM DETERMINES THE CHANGE OF LENGTH OF A BAR DUE
*    TO AXIAL TENSION OR COMPRESSION. THE BAR MAY BE A CONSTANT
*    THICKNESS, VARIABLE WIDTH RECTANGULAR SLAB, OR A SOLID CIRCULAR
*    ROD WITH VARIABLE DIAMETER. IN EITHER CASE THE SHAFT IS CENTRALLY

*    LOADED BY AN AXIAL FORCE.
*       THE VARYING WIDTH (OF THE SLAB) OR DIAMETER (OF THE ROD) MAY
*    BE DESCRIBED EITHER ANALYTICALLY AS   Y = A*E ^(B*X)   WHERE X IS THE
*    GEOMETRIC AXIS OF THE BAR, OR NUMERICALLY USING THE MAGNITUDE OF
*    Y AT EACH END OF  N SEGMENTS, MEANING N+1 VALUES.
*
*    INPUT:
*       THE USER IS PROMPTED FOR THE TOTAL BAR LENGTH, THE ELASTIC
*    MODULUS, AND THE AXIAL LOAD. THE USER IS THEN ASKED IF THE
*    BAR IS BOUNDED BY A KNOWN FUNCTION, AS WELL AS THE SHAPE OF ITS
*    X-SECTION. FOR THE CASE OF THE SLAB, THE UNIFORM THICKNESS IS
*    ALSO ASKED FOR... IF THE FUNCTION IS KNOWN, THE CONSTANTS ARE
*    THEN PROMPTED AND THE ENDPOINTS OF THE BAR ON THE X-AXIS INPUTTED;
*    ALTERNATELY, THE NUMBER OF SEGMENTS AND MEASURED HEIGHTS/DIAMETERS
*    MUST BE ENTERED.
*
*    OUTPUT:
*       THE TOTAL ELONGATION OF THE BAR IS DETERMINED AND PRINTED.
*
```

```
*    VARIABLES:
*       L,T,EM       ---  LENGTH,THICKNESS,ELASTIC MODULUS OF BAR
*       A,B          ---  CONSTANTS OF Y = A*E^(B*X)  GOVERNING BAR BOUNDA
*       X0,XN        ---  ENDPOINTS OF SHAFT ON X-AXIS
*       P            ---  CENTRALLY APPLIED AXIAL LOAD
*       AA(100)      ---  INDIVIDUAL SEGMENT HEIGHTS/DIAMETERS
*       AREA         ---  X-SECTIONAL AREA OF EACH SMALL INCREMENT
*       ANS          ---  DETERMINE IF USER HAS A KNOWN FUNCTION
*       TYPE         ---  DETERMINE BAR X-SECTION
*       DELTA        ---  UNIFORM BAR ELONGATION
*       LEN          ---  LENGTH OF INCREMENTAL ELEMENT
*
*******************************************************************
*******************************************************************
*                          MAIN PROGRAM
*******************************************************************
*******************************************************************
*
*    VARIABLE DECLARATION
*
     REAL I,T,L,EM,A,B,X0,XN,P,DELTA,AA(100),AREA,LEN
     INTEGER ANS,TYPE,NUM,J
*
*        USER INPUT PROMPTS
*
     PRINT*,'ENTER THE TOTAL LENGTH OF THE BAR (IN M OR INCHES):'
     READ*,L
     PRINT*,'ENTER THE ELASTIC MODULUS (IN PASCALS OR PSI) :'
     READ*,EM
     PRINT*,'ENTER THE UNIFORM AXIAL LOAD (IN NEWTONS OR LBS) :'
     READ*,P
     PRINT*,'PLEASE DENOTE THE BAR X-SECTIONAL SHAPE:'
     PRINT*,'ENTER  1--SLAB  ;   2--CIRCULAR ROD'
     READ*,TYPE
*
*        IF A SLAB, PROMPT FOR ITS THICKNESS
*
     IF (TYPE.EQ.1) THEN
        PRINT*,'ENTER THE THICKNESS OF THE SLAB (IN M OR INCHES):'
        READ*,T
     ENDIF
     PRINT*,'DO YOU KNOW THE FUNCTION DESCRIBING THE BAR?'
     PRINT*,'ENTER  1--YES  ;   2--NO'
     READ*,ANS
*
*        IF ANS EQUALS ONE, THE USER KNOWS FUNCTION. PROMPT
*        FOR CONSTANTS AND ENDPOINTS.
*
     IF (ANS.EQ.1) THEN
```



```
        PRINT*,'F(X)  = A*E^(B*X)'
        PRINT*,'ENTER A,B:'
        READ*,A,B
        PRINT*,'ENTER THE X-COORDINATE FOR BOTH ENDS OF THE BAR:'
        PRINT*,'(IN M OR INCHES):'
        READ*,X0,XN
*
     AREA = 0
     L=XN-X0
     LEN=L/50
     DO 20 I = X0,XN,LEN
        Y1=(A*(2.71828**(B*I)))*2
        Y2=(A*(2.71828**(B*(I + LEN))))*2
        Y=(Y1+Y2)/2
        IF(TYPE.EQ.1) THEN
           AREA=1/(Y*T) + AREA
        ELSE
           AREA=4/(3.14159*(Y**2)) + AREA
        ENDIF
  20 CONTINUE
*
*        IF ANS EQUALS TWO, THE USER DOES NOT KNOW FUNCTION.
*        PROMPT FOR NUMBER OF SEGMENTS AND MEASURED HEIGHTS/DIAMETERS.
*
     ELSE
        PRINT*,'ENTER THE NUMBER OF SECTIONS TO BE CALCULATED:'
        READ*,NUM
        IF(TYPE.EQ.1) THEN
           PRINT*,'ENTER THE HEIGHTS OF THE ENDS FOR SECTIONS 1 TO N:'
           PRINT*,'(IN M OR INCHES):'
        ELSE
           PRINT*,'ENTER THE DIAMETERS OF THE ENDS FOR SECTIONS 1 TO N:'
           PRINT*,'(IN M OR INCHES):'
        ENDIF
*
```

```
*          INPUT MEASURED HEIGHTS/DIAMETERS
*
         DO 30 J=1,NUM+1
            READ*,AA(J)
30       CONTINUE
*
         AREA = 0
         LEN = L/NUM
         DO 40 J = 1,NUM+1
            Y=(AA(J)+AA(J+1))/2
            IF(TYPE.EQ.1) THEN
               AREA = 1/(Y*T) + AREA
            ELSE
               AREA = 4/(3.14159*(Y**2)) + AREA
            ENDIF
40       CONTINUE
      ENDIF
*
*           DETERMINING THE ELONGATION OF THE LOADED BAR
*
      DELTA=(P*LEN*AREA)/EM
*
      PRINT 50,DELTA
*
50    FORMAT(2X,'THE DEFORMATION OF THE BAR IS:',F8.5,' (M OR IN.)')
*
      STOP
      END
```


A bar of variable solid circular cross section is bound by the curvature and extends from x=0 to x=180 in. It is subjected to axial tensile load of 100000 lb as shown in Fig. The material is steel for which E =30 x 10^6 lb/in2. Use the FORTRAN program to determine the elongation of bar.